Numerical Methods - 01 (18/7)

e.g. $\int_a^b e^x \, dx = \int_0^1 e^{(x \tan x \cos x)} \, dx$

e.g.

$$\begin{bmatrix} - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \end{bmatrix}$$ 10 x 10 matrix

e.g. $x^4 - 10x^3 + 10x^2 + 5 = 0$

Numerical methods :- the methods which provides to solve the real time computation of a problem such that the system will solve the problem with optimal time, those methods are Numerical methods.

Any problem can be defined as

$$y = f(x)$$

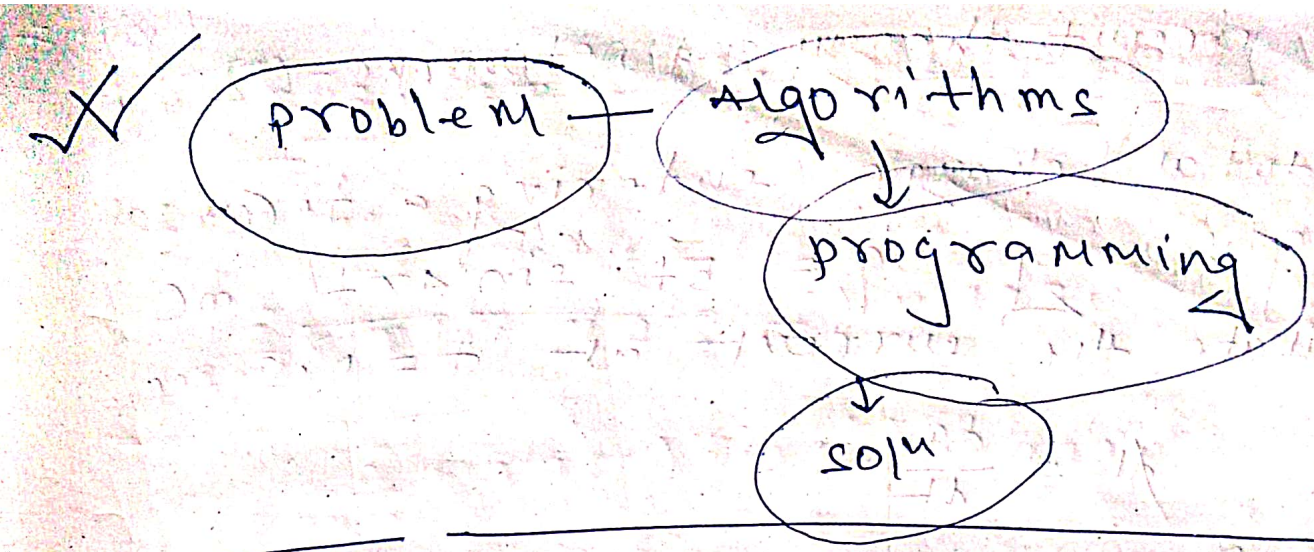
where x : set of inputs ✓

y : outcome ✓

f : procedure ✓

so the overall problem solving techniques by numerical methods are

1) error corrections & optimizations ✓
2) Numerical interpolations ✓
3) Numerical differentiation
4) Numerical Integration
5) systems for solution of unknowns
6) solutions for transdental equations
7) soln's for finite and ∞ series
8) Numerical diff. eqn soln
9) convergence techniques for these problems
10) problem soln's using c-programming language of these problems


problem
algorithms
programming
soln

Numerical Methods-02 (22/07/2024)

* Errors & error correction:-

$y = y_{original}$

$\hat{y} = y_{outcome}$

If for the given system $y = f(x)$, where x: set of inputs, f be the f/n, y: output

suppose, $y_{initial} = y_{original}$ and after computation the obtained value is $\hat{y}$.

If this → $|y - \hat{y}| = 0$, then exact solution

else if this → $|y - \hat{y}| < \varepsilon$ (any min value) then error occurred.

the nature of errors:

1) ~~Exact~~ Approximate error (Absolute error)

$$E_A = |y_T - y_A|,$$

$y_T \rightarrow$ True value of y

$y_A \rightarrow$ Approximate value of y.

2) Relative error:- (when there is a concept of Relativity)

$$E_R = \frac{E_A}{y_T}$$

y_T : Actual value of y.

3) Percentage error:-

$$E_P = E_R * 100$$

* the representation of Results:-

⇒ there are some techniques to optimize the floating pt representation:-

4) Truncation error : ∞-process is approximated $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots$

$x=1$ $e^1 = 2.5$

$e^1 = 2.718$

5) Rounding error —

$2.678 \rightarrow 2.68$

$= 2.68$

1) K- decimal places Representation.

e.g. $x = 4.57210101$

Reduce into 3 decimal places

$x = 4.572$

Reduce into 1 decimal place

$x_A = 4.5$

ii) if the k^{th} decimal places is value at greater than 5 (>5) then convert the number (x_A). where its k^{th} decimal value will be ↑ed by 1.

e.g. $x = 4.773$

$x_A = 4.8$ ($k=1$)

$x_A = 5$ ($k=0$)

iii) if the $(k+1)^{th}$ value is odd then convert x_A with its k^{th} value ↑ed by 1.

iii) If $(k+1)^{th}$ is ~~given~~ even then x_A with its k^{th} value will be same.

e.g. if $x = 4.563$

$x_A = 4.5$

** the assumption (ii) and (iii) will only be possible if the k^{th} decimal ~~decimal~~ value be only 5

e.g.5 ⇒ case-(i) $k \neq 5$ → $(k+1)^{th}$

$x_T = 4.7876$

convert 1 decimal place.

$x_A = 4.8$

case-iii) $x_T = 4.5876$ ($k=5$, $(k+1)^{th}$ even)

then $x_A = 4.5$ >

case-ii)

if $x_T = 4.5776$ ($k=5$, $(k+1)^{th} > 5$ and odd)

$x_A = 4.6$

Q) $x = 4.52671$

$x_A =$ } then find approximat-

2) significant figure :-

$x_T = 4.57891$ (6 sig. digit)

$x_A = 4.\overline{000}6$

significant digit : total no. of digits that represent that number

** A formula for error optimization :

for any given problem, there will be

1) a set of inputs :

$x : (x_1, x_2, \cdots x_n)$ ↳ vector / array

2) objective f/n (o.f)

$Z = a_1x_1 + a_2x_2 \cdots + a_nx_n$

a) $Z_{minimum} = \min_{Z} (Z)$

b) $Z_{maximum} = \max_{Z} (Z)$

3) Error :-

always minimization —

x_T x_A

$$E_A = |x_T - x_A|$$

do
{ apply 0.F 'z' to find z_A $z = z_A$
} while($|z - z_A| < \varepsilon$); $\varepsilon \to 0.01$

$z = 11.72$
$z_A = 10.12$ } diff = $1.60 < 0.01$ (X)

Next iteration :-

$z_A = 11.37$

$0.41 < 0.01$ (X)

:

10 times iterated

$\Rightarrow$ so: $|z - z_A| < \varepsilon$ → if this ↑ed then iteration ↓ed

↓ed then iteration ↑ed more accurate Ans.

** with setting the value of ε to minimize the errors between x_T and x_A, if value of ε is less then less iteration is req. and nature of correctness will be minimum.

conversely :-

If the ↑ing, the decimal places of 'ε' both the correctness and iteration ↑es while ↑es the Time complexity.

Assignments :-

1) convert the following no.s into 3 decimal places :-

4.97123 → 4.971

7.91258 → 7.913

8.11921 → 8.119

2) convert the given no.s into 5 ~~decimal places~~ significant figures

4.97123 → 4.9712

7.91258 → 7.9126

8.11921 → 8.1192

3) compute E_A, E_R, E_P for each conversion (given no.s)

4) compute the E_A, E_R, E_P b/w the converted decimal places and significant figures.

5) WAP in 'C' to perform these above objectives :-

Digital Logic

~~Fraction~~ ~~Logic~~ ——— frac $(.D_2D_1)$

$D_3D_2D_1$ (2nd) ----> $16^{-1} \times D_2 + 16^{-2} \times D_1$

$16^2 \times D_3 + 16^1 \times D_2 + 16^0 \times D_1$

H → D
O → D
B → D

D → H
D → O
D → B

R

O → D
O → H (via B)
O → B (3 bit)

H → D
H → O (via B)
H → B (4 bit)

B → D
B → O (3 bit)
B → H (4 bit)

Numerical Methods - 03 (25/7)

* Numerical Interpolation :-

Lets suppose there are n given pts

$(x_1, y_1), (x_2, y_2) \cdots (x_n, y_n)$

In 2D, such that using these pts a f/n $f(x,y)$ can be formed, the derived f/n $f(x,y)$ have limit to a particular pt. with satisfying the property of continuity.

There are 4 well known techniques for numerical Interpolation.

1) Langrange interpolan formulae.
2) Newton divided difference I.F
3) Newton forward I.F
4) Newton Backward I.F

** the L.I.F, N.D.D.I.F, ~~NFIF~~, ~~NBIF~~ are based on unequal width of given x

** NFIF, NBIF : Based on equal interval of x

equal

x :	2	4	6	8	10	12

unequal

x :	2	-1	8	6	-4	1

** the soln from N.F.I.F & NBIF can also be obtained from LIF & NDDIF. but the conversely statements are not true;

Langrange Interpolation formula

Method
technique
e.g.
programme

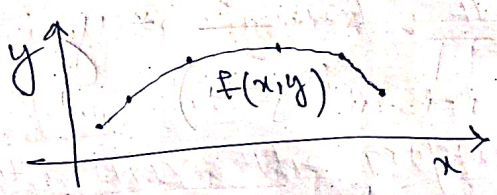
⇒ lets suppose there are n given pts

$(x_1, y_1), (x_2, y_2), (x_3, y_3) \cdots (x_n, y_n)$

Step-I :- Arrange x_i, y_i in column wise

x_1	y_1
x_2	:
x_3	:
:	:
x_n	y_n

Step-II :- (Apply the techniques



$$f(x,y) = \frac{(x-x_2)(x-x_3)\cdots(x-x_n)}{(x_1-x_2)(x_1-x_3)\cdots(x_1-x_n)} \times y_1 +$$

V.V.I

$$\frac{(x-x_1)(x-x_3)\cdots(x-x_n)}{(x_2-x_1)(x_2-x_3)\cdots(x_2-x_n)} \times y_2 + \cdots$$

$$+ \frac{(x-x_1)(x-x_2)\cdots(x-x_{n-1})}{(x_n-x_1)(x_n-x_2)\cdots(x_n-x_{n-1})} \times y_n$$

$$= \frac{\prod_{i=2\to n,\, i\neq 1}(x-x_i)}{\prod_{i=2\to n}(x_1-x_i)} \times y_1 + \frac{\prod_{i=1\text{ to }n,\, i\neq 2}(x-x_i)}{\prod_{i=1\text{ to }n}(x_2-x_i)} \times y_2 + \cdots +$$

$$\frac{\prod_{i=1\cdots n,\, i\neq n}(x-x_{n-1})}{\prod_{i=1\cdots n,\, i\neq n}(x_n-x_i)} \times y_n$$

$$= \sum_{j=1}^{n} \frac{\prod_{i=1\cdots n,\, i\neq j} (x-x_i)}{\prod_{i=1\cdots n,\, i\neq j} (x_j-x_i)} * y_j$$

$$= \sum_{\substack{j=1 \\ i\neq j}}^{n} \frac{\prod_{i=1\cdots n} (x-x_i)}{\prod_{i=1\cdots n} (x_j-x_i)} * y_j$$

Time complexity of L.I.F $O(n^2)$

e.g.s Find the Interpolation fn from the given data:

⇒

x	1	2	1
y	0	2	4

⇒ step 1 :

x	y
$x_1 \leftarrow 1$	$0 \rightarrow y_1$
$x_2 \leftarrow 2$	$2 \rightarrow y_2$
$x_3 \leftarrow 4$	$4 \rightarrow y_3$

$$f(x,y) = \frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)} \times y_1 + \frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)} \times y_2 + \frac{(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)} \times y_3$$

$$= \frac{(x-2)(x-4)}{(1-2)(1-4)} \times 0 + \frac{(x-1)(x-4)}{(2-1)(2-4)} \times 2 + \frac{(x-1)(x-2)}{(4-1)(4-2)} \times 4$$

$$= -(x-1)(x-4) + \frac{2}{3}(x-1)(x-2)$$

$f = (\ )x^2 + (\ )x + c$

find the value at $y_{4.9}$ that is put $x = 4.9$

Assignments :- ② Find $f(11.5)$ where the given data are:

x	10	21	9
y	31	2	−4

$(71.73*)$

③ $f(0.212)$, upto 6 sig. fig.

x	2	3	0	5	2
y	−1	2	−1	3	−1

ignoring

$y = (-1.766552)$

-1.7666

(4) WAP in C to implement Langrange Interpolation formulae with keeping the data upto 7th order.

$$\begin{Bmatrix} x_1, x_2 \cdots x_7 \\ y_1, y_2 \cdots y_7 \end{Bmatrix}$$

⇒ Hints:- 2 Array

```
double factor = pow(10, n);
double scale  = pow(10, sig - floor(log10(fabs(value))) - 1);
return round(value * scale) / scale;
      (value * factor) / factor);
```

```
* dbl LIF(x[], y[], xp, n) order
    dbl yp = 0;
  for( i = 0; < n; i++) {   if(i ≠ j) {
    dbl L = 1;
  for( j = 0; < n; j++) {
      L = L * (xp - x[j]) / (x[i] - x[j]);
    yp = yp + L * y[i];
  }
  return yp;
}
```

Numerical Methods - 04 (05/08/24)

(2) Newton's divided difference formulae (NDDIF)

for the given n pts

$(x_1, y_1), (x_2, y_2), \dots (x_n, y_n)$

where $x_1, x_2 \dots x_n$ does not have any common difference then according to NDDIF

V.V.I

$$f(x) = y_1 + (x - x_1) f[x_1, x_2] + (x - x_1)(x - x_2) f[x_1, x_2, x_3] + \dots$$
$$(x - x_1)(x - x_2) \dots (x - x_{n-1}) \times f[x_1, x_2 \dots x_n]$$

where $f[*]$ is the divided difference of the corresponding pt derived from the divided difference table.

the following is the divided difference table :-

x	y	$f[x_1,x_2]$	$f[x_1,x_2,x_3]$	$f[x_1,x_2,x_3,x_4]$
x_1	y_1	$\frac{y_2-y_1}{x_2-x_1}$	$\frac{y_3-y_1}{x_3-x_1}$	$\frac{y_4-y_1}{x_4-x_1}$
x_2	y_2	$\frac{y_3-y_2}{x_3-x_2}$	$\frac{y_4-y_2}{x_4-x_2}$	$\frac{y_5-y_2}{x_5-x_2}$
x_3	y_3	$\frac{y_4-y_3}{x_4-x_3}$	⋮	⋮
x_4	y_4	⋮		
x_5	y_5			
⋮	⋮			
x_n	y_n	$\frac{y_n-y_{n-1}}{x_n-x_{n-1}}$	$\frac{y_n-y_{n-2}}{x_n-x_{n-2}}$	$\frac{y_n-y_{n-3}}{x_n-x_{n-3}}$

$\cdots \quad f[x_1,x_2,\cdots x_n] = \frac{y_n-y_1}{x_n-x_1}$

$\vdots \quad \vdots \quad n-1$

$\vdots \quad \vdots \quad n-2$

$\vdots \quad \vdots \quad 1$

Pascal's triangle ↑ nC

* Steps for solution :-

1) the given set of pts
2) the arrangement of given set of pts into tabular form
3) compute the divided difference table
4) By using the divided diff table compute the function.

time complexity

$n + (n-1) + (n-2) \cdots + 1 + n$

$= O(n^2)$

e.g.:

suppose the given pts are, now apply NDDIF, to obtain the f/n $f(x)$ and find $f(1.3)$

x	y	$f[x_1,x_2]$	$f[x_1,x_2,x_3]$	$f[x_1,x_2,x_3,x_4]$
4.	(11).	$\frac{9-11}{3-4} = \frac{-2}{-1}$	$\frac{2-2}{0-4} = \frac{0}{-4}$	$\frac{-1-0}{5-4} = -1$
3.	9.	$\frac{3-9}{0-3} = \frac{-6}{-3}$	$\frac{0-2}{5-3} = \frac{-2}{2} = -1$	
0.	3.	$\frac{2-3}{5-0} = \frac{-1}{5}$		
5.	2.			

$= -1.1$

Next Page

$f(x) = y_1 + (x-x_1) f[x_1,x_2] + (x-x_1)(x-x_2) f[x_1,x_2,x_3] + (x-x_1)(x-x_3)(x-x_2) f[x_1,x_2,x_3,x_4]$

$= 11 + (1.3-4) \times 2 + (1.3-4)(1.3-3) \times 0$

x	y
4	11
3	9
0	3
5	2

apply NDDIF at $f(1.3)$ } example 1

Note:

Revised NDDIF

x	y	$f[x_1, x_2]$	$f[x_1, x_2, x_3]$
x_1	y_1	$\frac{y_2 - y_1}{x_2 - x_1}$	$\frac{f[x_3, x_2] - f[x_2, x_1]}{x_3 - x_1}$
x_2	y_2	$\frac{y_3 - y_2}{x_3 - x_1}$	$\frac{f[x_4, x_3] - f[x_3, x_2]}{x_4 - x_2}$
x_3	y_3	⋮	⋮
x_4	y_4		$\frac{f[x_n, x_{n-1}] - f[x_{n-1}, x_{n-2}]}{x_n - x_{n-1}}$
⋮	⋮		
x_n	y_n		

$f[x_1, x_2, x_3, x_4]$

$$\frac{f[x_4, x_3, x_2] - f[x_3, x_2, x_1]}{x_4 - x_1}$$

⋮

$$\frac{f[x_n, x_{n-1}, \ldots x_2] - f[x_{n-1} \ldots x_1]}{(x_n - x_1)}$$

Numerical Methods - 05

22/08/24

* (3) Newton's forward interpolation formula with equal common difference:

For the n given pts

$(x_1, y_1), (x_2, y_2), (x_3, y_3) \cdots (x_n, y_n)$

where $\boxed{p = \left(\frac{x - x_1}{h}\right)}$, h = common difference.

Hence the formula is written as -

$$f(x) = y_1 + p(\Delta y_1) + \frac{p(p-1)}{\lfloor 2}(\Delta^2 y_1) + \frac{p(p-1)(p-2)}{\lfloor 3}(\Delta^3 y_1) + \cdots + \frac{p(p-1)\cdots(p-n+1)}{\lfloor n}\Delta^n y_1$$

where, (x_1, y_1) be the initial pt of the interpolation.

Eg. the given pts are

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
1891	46	20	-5	+2
1901	66	15	-3	
1911	81	12		
1921	93			

(0,0) (1,0) (2,0) (0,1) (0,2) (0,3)

* For the given 'n' pts there will be Δ^{n-1} order of difference.

$f(x)$

$p = \frac{x - x_4}{h}$

$= \frac{x - 1891}{10}$

$= 46 + \left(\frac{x-1891}{10}\right)20 + \frac{p(p-1)(-5)}{\lfloor 2} +$

$\frac{p(p-1)(p-2)}{\lfloor 3} \times 2$

Eg 2)

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
11	4	-2	5	-7	+4
13	2	3	-2	-3	
15	5	1	-5		
17	6	-4			
19	2				

$f(x) = y_1 + p\Delta y_1 + \frac{p(p-1)}{\lfloor 2} \Delta^2 y_1 + \frac{p(p-1)(p-2)}{\lfloor 3} \Delta^3 y_1$

$+ \frac{p(p-1)(p-2)(p-3)}{\lfloor 3} \cdot \Delta^4 y$

here $p = \frac{x - x_4}{h} = \frac{x-11}{2}$

at $f(100) = ?$ $\boxed{x = 100} \Rightarrow p = \frac{100-11}{2} = \frac{89}{2}$

(4) * Newton's Backward Interpolation formula :-

for the given n pts $(x_1, y_1) \cdots (x_n, y_n)$

where $p = \frac{x - x_n}{h}$, h : common difference.

the formula is as follows :-

$f(x) = y_n + p\nabla y_n + \frac{p(p+1)}{\lfloor 2} \nabla^2 y_n + \cdots$

$+ \frac{p(p+1)(p+2)\cdots(p+n-1)}{\lfloor n} \nabla^n y_n$

Eg 3)

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$
1891	46	20	-5	2
1901	66	15	-3	
1911	81	12		
1921	93			

$f(x) = 93 + p \cdot 12 + \frac{p(p+1)}{\lfloor 2} \times (-3)$

$+ \frac{p(p+1)(p+2)}{\lfloor 3} \times (2)$

here $p = \frac{x - x_n}{h} = \frac{x - 1921}{10}$

(In question x will be given : 100) or anything

* Assignments :-

1) Lets suppose the given pts are

x	y
10	21
20	49
30	32
40	50
50	102

use both N.F.I & N.B.I to find the value of $f(120)$ in both the cases.

compare the result of this two results. if there is some error find (E_R) and (E_P)

2) C programming for NFI, NBI :-

⇒

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
10	21	28	-45	80	-81
20	49	-17	35	-1	
30	32	18	34		
40	50	52			
50	102				

∇y | $\nabla^2 y$ | $\nabla^3 y$ | $\nabla^4 y$

$h = 20-10 = 10$, $x = 120$, $p = \frac{120-50}{10} = 7$

$p = \frac{120-50}{10} = 7$

don't need to calculate this

NFI: $f(x) = y_1 + p \cdot \Delta y_1 + \frac{p(p-1)}{2!}\Delta^2 y_1 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_1 + \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_1 = 28 + (11 \times 28) + \frac{11 \times 10}{2} \times (-45) + \frac{11 \times 10 \times 9}{6} \times 80 + \frac{11 \times 10 \times 9 \times 8}{24} \times (-81) = -15676$

NBI :- $f(x) = y_n + p \cdot \nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \frac{p(p+1)(p+2)(p+3)}{4!}\nabla^4 y_n = 102 + (7 \times 52) + \frac{7 \times 8}{2} \times 34 + \frac{7 \times 8 \times 9}{6}(-1) + \frac{7 \cdot 8 \cdot 9 \cdot 10}{24} \times (-81) = -15676$

$E_R = 0$, $E_P = 0$

* Numerical Integration :-

Numerical methods provides the solution of finding the values of definite integrals.

there are some methods :-

1) Trapizoidal method for Numerical Integration :-

⇒ e.g. for the given integration I

$$I = \int_a^b f(x)\,dx$$

, the value of this integration, according to trapizodal method is

$$I = \frac{h}{2}\left[f(a) + f(b) + 2\{\text{sum of all other terms}\}\right]$$

let 'n' be the no. of terms

if $n = 10$

$f(1) = f(a)$ } other times will be
$f(12) = f(b)$ } $f(2)$ --- $f(11)$

$$h = \frac{b-a}{n}$$

Eg: Find the value of integration

$$I = \int_0^1 e^x \tan x \, dx, \text{ where } n = 12$$

$$\Rightarrow h = \frac{1-0}{12} = \frac{1}{12} = 0.08333 \approx 0.08$$

$$\Rightarrow I = \frac{0.08}{2}\left[f(0) + f(1) + 2\{f(x_2) + f(x_3) + \dots f(x_{11})\}\right]$$

no. of terms = 12

$x_1 \dots\dots x_{12}$

$\downarrow \quad\quad \downarrow$

$a \quad\quad b$

* for the other term we have to compute the table.

n	x	f(x)	f(x)
1	0	f(0)	f(x1)
2	0+h	f(0+h)	f(x1+h)
3	0+2h	f(0+2h)	f(x1+2h)
⋮	⋮	⋮	⋮
10	0+9h	f(0+9h)	⋮
⋮	⋮	⋮	f(x1+(n-1)h)
13	1	f(1)	

Eg: solve

$$I = \int_0^1 e^x \tan x \, dx \quad \text{for } n = 4$$

a → 0, b → 1

n	x	f(x)	
1	0	f(0)	0
2	0.25	f(0.25)	0.327
3	0.5	f(.5)	0.9
4	0.75	f(.75)	1.97
5	1	f(1)	4.23

f(x) = $e^x \tan x$

tan(.5) → radian

shift + mode → (4)

till 1 comes

$$h = \frac{b-a}{n} = \frac{1-0}{4} = 0.25$$

$$I = \left(\frac{0.25}{2}\right)\left[0 + 4.23 + 2 \times (\quad)\right] = ?$$

* Assignment :-

1) Find the value of $\int_{-5}^{5} e^x \, dx$ using conventational method and Trapizodial method.

given that, $h = 1$ step-1

$$h = \frac{b-a}{n} = \frac{10}{n} = 1$$

$$\Rightarrow n = 10$$

⇒ compare both the results (conventational & Trapizodal)

If there is an error find E_R & E_P

2) Implement of C programming for the soln of this Integration Method.

$$I = \int_{-5}^{5} e^x \, dx$$

** Time complexity for Trapizodal Method. O(n)

$$\int_{-5}^{5} e^x \, dx = e^5 + e^{-5} = 148.4198$$

h=1 n=10

n	x	f(x)
1	1	e
2	2	e^2
3	3	e^3
4	4	e^4
5	5	e^5
6	6	e^6
7	7	e^7
8	8	e^8
9	9	e^9
10	10	e^{10}

$f(x) = e^x$

$f(-5) = e^{-5}$

$I = \frac{1}{2}[148.413 + 1.0067 + 2(e^2 + e^3 + e^4 + e^5 + e^6 + e^7 + e^8 + e^9)]$

$= 0.5$

$f(x) = e^x$

n	x	f(x)
0	-5	e^{-5}
1	-4	e^{-4}
2	-3	e^{-3}
3	-2	e^{-2}
4	-1	e^{-1}
5	0	e^0
6	1	e
7	2	e^2
8	3	e^3
9	4	e^4
10	5	e^5

$I = \frac{1}{2}[148.4198 + 2(e^{-4} + \cdots e^4)]$

$= \frac{1}{2}[160.5722]$

$E_a = 12.1659$

$E_r = \frac{12.1659}{148.4198} = 0.08196$

$E_p = 0.08196 \times 100\% = 8.196\%$

* Numerical Integration using Simpson's 1/3rd rule:-

2)

$$I = \int_a^b f(x) \, dx \quad ; \quad h = \frac{b-a}{n}$$

n = No. of iteration

$$I = \int_a^b f(x) \, dx = \frac{h}{3}[(\text{First term} + \text{last term}) + 4(\text{sum of all odd terms}) + 2(\text{sum of all even terms})]$$

a ——— b

x_0 ——— x_n

$f(x_0)$ ----- $f(x_n)$

y_0 ——— y_n

$$I = \frac{h}{3}[y_0 + y_n + 4(y_1 + y_3 + y_5 + \cdots + y_{n-1}) + 2(y_2 + y_4 + y_6 + \cdots + y_{n-2})]$$

Eg:

$$\int_0^1 e^{-x^2} \, dx \quad , \quad n = 8$$

$$h = \frac{1-0}{8} = 0.125$$

n	x	$y = e^{-x^2}$	
0	0	1	y_0
1	0.125	–	y_1
2	0.250	–	y_2
3	0.375	–	y_3
4	0.500	–	y_4
5	0.625	–	y_5
6	0.750	–	y_6
7	0.875	–	y_7
8	1.000	e^{-1}	y_8

$x_4 = x_{4-1} + h$

.375 .125 .500 .625 .125 .750 .875 .125 1.00

$$I = \frac{0.125}{3}\left[1 + A + 4(y_1 + y_3 + y_5 + y_7) + 2(y_2 + y_4 + y_6)\right]$$

③ **Simpson's 3/8 rule :-**

$$I = \int_a^b f(x)\,dx \qquad h = \frac{b-a}{n}, \quad n = \text{no. of iteration}$$

v.v.I

$$I = \frac{3h}{8}\left[(y_0 + y_n) + 3(y_1 + y_2 + y_4 + y_5 + \cdots y_{n-1}) + 2(y_3 + y_6 + y_9 + y_{12} \cdots y_{n-3})\right]$$

$$= \frac{3h}{8}\left[\left(\text{First term} + \text{last term}\right) + 3\left(\text{sum of all terms except multiple of 3}\right) + 2\left(\text{sum of all the terms i.e. multipliers of 3}\right)\right]$$

* As comparison trapezodal, simpson's $\frac{1}{3}$rd & simpson's $\frac{3}{8}$ rule :-

it has been observed that the error minimization is more specific with the convergence of the function.

Hence, the obtained error among these methods is

$$E = \left\{\left(-\frac{1}{90}\right)\left\{\left(\frac{b-a}{2}\right)^5\right\} f^4(\xi)\right\}$$

* ξ is some number b/w a & b but very close to a or b.

④ **Boole's Rule :-**

$$I = \int_a^b f(x)\,dx \qquad h = \frac{b-a}{n}$$

v.v.I

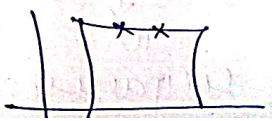
$$I = \frac{3h}{45}\left[7(y_0 + y_n) + 32(y_1 + y_3 + y_5 + y_7 \cdots) + 12(y_2 + y_6 + y_{10} + \cdots y_{14} + \cdots) + 14(y_4 + y_8 + y_{12} + \cdots)\right]$$

(1st term, last term)

2, 6, 10, 14 ---

4, 8, 12, 16 ---

Break even point → when there is a break pt/ discontinuity in a function



5) **Weddle's Rule :-**

$$I = \int_a^b f(x)\,dx, \qquad h = \frac{b-a}{n}$$

$$I = \frac{3h}{10}\left[(y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6) + (y_6 + 5y_7 + y_8 + 6y_9 + y_{10} + 5y_{11} + y_{12}) + (y_{12} + 5y_{13} + y_{14} + 6y_{15} + y_{16} + 5y_{17} + y_{18}) + \dots\right]$$

1 5 1 6 1 5 1

151615 1

* Assignments :-

1) C-programming for ② ③ ④ ⑤ rule. for the below problem

2) solve this $\int_0^1 e^x \tan x\,dx$ with $n = 8$ using ② ③ ④ ⑤ & ①

3) compare the result of this ⑤ methods with error terms, if suppose there are E_p E_p.

are 5 out comes using 5 methods, compare one methods with other. $(5C_2)$ → 10 comparison

⇒ For $\int_0^1 e^x \tan x\,dx$ — VSC code.

Simpson's 1/3rd : 1.265445

" 3/8 : 1.265524

Boole's rule : 1.265385

weddle's rule : 1.265382

Trapizodal's rule :- 1.272619

Method 1	Method 2	E_R	E_P
Trapizodal	1/3rd	0.007174	0.5638 -/-
"	3/8	0.007095	0.5575 -/-
"	Boole's	0.007234	0.5685 -/-
"	W	0.7237	0.5688 -/-
"		0.000079	0.0062 -/-
1/3rd	3/8	0.000060	0.0047 -/-
1/3rd	Boole's	0.000063	0.0050 -/-
1/3rd	W	0.000139	0.0110 -/-
3/8	Boole's	0.000142	0.0112 -/-
3/8	W	0.000003	0.0002 -/-
B	W		

Trapizodal → highest error

W & B → high accuracy

1/3 & 3/8 → moderately accurate.

Assignment solution

Numerical Methods - 07 (02/9/24)

* Numerical solution for transcendental equation :-

TRANSCENDENTAL

- to find the root of the given equation :-

$f(x)$, $f(x)$ at $x = c$, $\int_a^b f(x) = ?$. $f(a) = 0$

$x^x - x + 3 = 0$

$\tan x - \sin x = 0$

the transcedental eqn means finding root (solution) of the given eqn $f(x) = 0$, such that the f/n $f(x)$ will be defined with any composition of logarithmic, trigonometrical, exponential, Algebric, etc.

etc:

$x^x e^{\tan x} + \sin x = 0$

$x \log x + e^x = 0$

1) the solution of Transcedental equation using ~~Brige~~ Bisection Method :-

Bisection

BISECTION

Algorithm for Bijection method :-

step 1 :- consider $f(x) = 0$, is the given eqn.

step 2 :- find the interval $[a, b]$, where the root of $f(x) = 0$ lies

step 3 :- if $f(a) * f(b) < 0$ then root $x \in [a, b]$

step-4 :- suppose $x = \frac{a+b}{2}$ if $x \in [a, b]$

step-5 :- if $f(b) * f(x) < 0$, then change $a \leftarrow x$

if $f(a) * f(x) < 0$ " " $b \leftarrow x$

step 6 :- continue step 4 to 6 until $f(x) = 0$

* for every method not only bijection step ① ② ③ will be remain the same only ④ ⑤ ⑥ will be different

eg ① $x^3 - x + 1 = 0$ see the next example

⇒ 1) $f(x) = x^3 - x + 1$

2) $f(0) = +1$ $f(2) = +$

$f(1) = +1$ $-$

sign change

root lies b/w 1 & 2

e.g. $x^4 - 2x - 1 = 0$

$f(0) = -1$, $f(1) = 1 - 2 - 1 = -2 < 0$

$f(2) = 16 - 8 - 1 = 7 > 0$

solution lies b/w 1 & 2

If $x^4 + 2x + 1 = 0$

then check $f(-1) =$ — ✓

$f(-2) =$ — ✓

from e.g. (1) V.V.I

Find Numerical table

a	f(a)	b	f(b)	$x = \frac{a+b}{2}$	f(x)
1	$-1 < 0$	2	$5 > 0$	$x = \frac{1+2}{2} = 1.5$	$f(1.5) = 0.875 > 0$
1	$-1 < 0$	1.5	$0.875 > 0$	$x = \frac{2.5}{2} = 1.25$	$f(1.25) = -0.2 < 0$
1.25	$-0.2 < 0$	1.5	> 0	$x = \frac{1.25+1.5}{2} = 1.375$	$f(1.375) > 0$
1.25	< 0	1.375	> 0	$x = 1.312$	< 0
1.312	< 0	1.375	> 0	$x = 1.345$	> 0
1.312	< 0	1.345	> 0	$x = 1.327$	> 0
				$x = 1.319$	

sign same

Now, Interchange

Exam type Q's

Apply Bisection Method to find

5th iteration $x = ?$

or

$x = ?$ at 1 decimal places

↓ exact soln

** the roots of the $f(x) = 0$ will depend the number of iterations, the no. of decimal places & also the number of significant figures.

*** the more exact soln needs more iterations with more decimal places.

Assignment: C programming for

- Decimal places
- 2 decimal "
- 3 " "

→ $f(x) = 0$: $f(1)$ $f(2)$ $f(3)$ $f(-1)$ $f(-2)$ $f(-3)$ $f(0)$ } hit & trial [a, b]

→ Table

Numerical Methods-08

(2) Iterative Method :-

[soln for Transcendental equation]

this method · find the solutions of $f(x) = 0$, with finding interval $[a, b]$ whereas the soln needs the test of convergence for this problem is given as · $|\phi'(x)| < 1$, where

$x = \varphi(x)$ at $(x = a) \rightarrow (x_0) = \frac{a+b}{2}$

⇒ eg. $x^3 - x - 1 = 0$

$f(x) = x^3 - x - 1$

$[a, b] = [1, 2]$

i) $f(0) = -1$ ⊖
$f(1) = -1$ ⊖
$f(2) = +$ ⊕
} $[1,2]$ → sign change

$x^3 - x - 1 = 0$

$\varphi(x) = x$

ii) ⇒ possibilites:

$x = x^3 - 1 = \varphi_1(x)$

$x = (1+x)^{\frac{1}{3}} = \varphi_2(x)$

$x = \frac{1}{x^2 - 1} = \varphi_3(x)$

⇒ $\varphi_1'(x) = 3x^2$

$\varphi_2'(x) = \frac{1}{3}(1+x)^{-\frac{2}{3}}$, $\varphi_3'(x) = \frac{-2x}{(x^2-1)^2}$

⇒ at $x = a = 1$

$|\varphi_1'(x)| = 3x$ ✗ , $|\varphi_2'(x)| < 1$ ✓ , $|\varphi_3'(x)| > 1$ ✗

iii) update the solution for $x_{n+1} = \varphi(x_n)$

$\varphi_2(x) = (1+x)^{\frac{1}{3}}$

n	x_n	$x_{n+1} = \varphi(x_n) = (1+x_n)^{\frac{1}{3}}$
0	$x_0 = 1$	$x_1 = (1+1)^{\frac{1}{3}} = 2^{\frac{1}{3}} = 1.26$
1	$x_1 = 1.26$	$x_2 = (1+1.26)^{\frac{1}{3}} = 2.26^{\frac{1}{3}} = 1.308$
2	$x_2 = 1.308$	$x_3 = (1+1.30)^{\frac{1}{3}} = (2.308)^{\frac{1}{3}} = 1.317$
3	1.317	$(2.317)^{\frac{1}{3}} = 1.319$

for 2 decimal places

ans: $x = 1.31$

(3) Newton Raphson Method

i) given $f(x) = 0$:

2) find the interval $[a, b]$

3) find $f'(x)$

4) Iteration table

$\left\{ \begin{array}{l} f(x) = x^3 - x - 1 \\ f'(x) = 3x^2 - 1 \end{array} \right\}$ another example

→ table : another example

n	x	f(x)	f'(x)	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
0	1	0.875	5.75	$1 - \frac{0.875}{5.75} = 1.3478$
1	1.3478	0.10068	4.449	1.3252
2	1.3252	0.0026	4.268	1.3247

any $\frac{a+b}{2}$.

④ Regula falsi method :-

It's the modified method of bijection & Newton Raphson method.

the bijection method bisects the soln b/w two interval $[x_1, x_2]$, whereas the Newton-Raphson method finds the solution with initial value & the value at this fn.

i) given $f(x) = 0$.

ii) find the interval $[a, b]$

iii) Iteration table

* $$x_{n+2} = \frac{x_n f(x_{n+1}) - x_{n+1} f(x_n)}{f(x_{n+1}) - f(x_n)}$$

for $n=0$,

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

n	x_0 (a)	$f(x_0)$	x_1 (b)	$f(x_1)$	x_2 **	$f(x_2)$	update
n=0	1	−1 < 0	2	5 > 0	1.66	−0.57 < 0	
	1.66	−0.57 < 0	2	5 > 0	1.25	−0.28 < 0	
	1.25	−0.28 < 0	2	5 > 0	1.29	+0.12 > 0	
	1.29	−0.12 < 0	2	5 > 0	1.31	−0.05 < 0	

⑤ Scant Method :-

It's the modify version of regula-farsi Method with update a value

$x_0 = x_1$
$x_1 = x_2$

n	x_0	$f(x_0)$	x_1	$f(x_1)$	x_2	update
	1	−1	2	5	1.66	$x_0 = x_1$, $x_1 = x_2$
	2	5	1.66	−0.57	1.25	$x_1 \to x_2$, $x_2 \to x_3$
	1.66	−0.57	1.25	−0.28	1.33	
	1.25	−	−	−		
	1.33					
	1.29	−	1.29	−	1.29	

now, $x_0 = x_1 = x_2$ (really same)

Numerical methods - 09 (23/09/2024)

* system of solution for linear equation

$x = 1$ $\quad 2x + 7y = 3$

$x + y = 2$ $\quad x_1 + 7x_2 + 3x_3 = 4$

if n var. $x_1, x_2 \dots x_n$

n eqn: ----

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$
$$\vdots$$
$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$$

If there are 'n' variables for 'm' equations.

i) $n > m$ $\left(\begin{matrix} 3x_1 + 7x_2 + 5x_3 = 2 \\ 2x_1 + 11x_2 + 13x_3 = 4 \end{matrix}\right)$ → NO SOLN

ii) $n = m$ (there will be unique soln)

iii) $n < m$ $\left(\begin{matrix} 3x_1 + 2x_2 = 4 \\ 2x_1 + 5x_2 = 7 \\ 5x_1 + 9x_2 = 11 \end{matrix}\right)$ → more than one soln

(*) If $n > m$: there will be no solution

(**) → Simplex Method

→ LPP (linear programming problems)

→ QPP (Quadratic)

→ IPP (Integer)

* For the objective of the numerical soln, we will consider $n = m$ possibilities

i.e. the no. of var = no. of eqn

the following methods will be applied for this type of problems :-

1) Gauss Elimination method
2) Gauss Jordan
3) Lower, upper, triangular matrix factorization method (LU factorization)

(non iterative method)

4) Gauss Jacobian method
5) Gauss seidel

(Iterative method)

(1) Gauss Elimination :- suppose there is a given system of linear eqn.

⇒ $Ax = b$

A → coefficient matrix, b → bias vector

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

x → unknown

Step-1

I) First define Augmented Matrix

$$[A/b]$$

$$\Rightarrow \left[\begin{array}{ccc|c} a_{11} & \cdots & a_{1n} & b_1 \\ a_{21} & & & b_2 \\ \vdots & & & \vdots \\ a_{n1} & \cdots & a_{nn} & b_n \end{array}\right]$$

II) Reduce the augmented matrix (AU Matrix) in such a way that

$$\left[\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & c_1 \\ 0 & a_{22} & \cdots & a_{2n} & c_2 \\ 0 & 0 & a_{33} & & \vdots \\ \vdots & & & & \\ 0 & 0 & 0 \cdots & a_{nn} & c_n \end{array}\right]$$

III) Find the soln from the reduced AU matrix

e.g. Find the soln for the given problem

$$x + 2y + 4z = 2$$
$$2x + y + 3z = 3$$
$$3x + 11y + 13z = 8$$

$$\Rightarrow AU = \left[\begin{array}{ccc|c} 1 & 2 & 4 & 2 \\ 2 & 1 & 3 & 3 \\ 3 & 11 & 13 & 8 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 4 & 2 \\ 0 & -3 & -5 & -1 \\ 0 & 5 & 1 & 2 \end{array}\right]$$

$R_3 \to 3R_3 + 5R_2$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 4 & 2 \\ 0 & -3 & -5 & -1 \\ 0 & 0 & -22 & 1 \end{array}\right]$$

→ lower triangular matrix

$$\Rightarrow x + 2y + 4z = 2$$
$$-3y + 5z = -1$$
$$-22z = 1 \Rightarrow z = -\frac{1}{22}$$

$y = (\quad)$

$x = (\quad)$

In Gauss Jordan: $\left[\begin{array}{ccc|c} 1 & 0 & 0 & - \\ 0 & 1 & 0 & - \\ 0 & 0 & 1 & - \end{array}\right]$ → pivotal matrix

↓ next class

Numerical Methods-10 (30/9)

soln of linear eqn

1) Gauss elimination $\begin{bmatrix} 1 & \cdot & \cdot \\ 0 & 1 & \cdot \\ 0 & 0 & 1 \end{bmatrix}$
2) Gauss Jordan $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
3) **LU - factorization** :

soln of the given system of linear eqn

$$\boxed{Ax = b} \quad -(1)$$

A : co-efficient Matrix which is further factorized

$$\boxed{A = LU} \quad -(11)$$

where L : lower triangular matrix
U : upper "

$$\begin{bmatrix} l_{11} & 0 & 0 & \cdots & 0 \\ l_{21} & l_{22} & 0 & \cdots & 0 \\ \cdot & & & & \\ l_{n1} & \cdots & & & l_{nn} \end{bmatrix} \quad \begin{bmatrix} u_{11} & \cdots & \cdots & u_{1n} \\ 0 & & & \\ 0 & & & \\ \cdot & & & \\ 0 & & & u_{nnn} \end{bmatrix}$$

for the soln of linear eqns the steps are :

$$\Rightarrow Ax = b \quad -(1)$$

$$\Rightarrow \underbrace{LU}_{y} x = b \quad -(111)$$

$$\Rightarrow \underbrace{Ly}_{} = b \quad -(1V)$$

$$Ux = y \quad -(V)$$

$$\begin{bmatrix} 2 \\ 4 \\ 5 \end{bmatrix}$$

* For this soln the $L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \\ l_{41} & l_{42} & l_{43} & 1 \end{bmatrix}$

Eg: $x_1 + x_2 + x_3 = 1$
$3x_1 + x_2 - 3x_3 = 5$
$x_1 - 2x_2 - 5x_3 = 10$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & -3 \\ 1 & -2 & -5 \end{bmatrix} \Rightarrow LU = A$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & -3 \\ 1 & -2 & -5 \end{bmatrix}$$

$$\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12}+u_{22} & l_{21}u_{13}+u_{23} \\ l_{31}u_{11} & l_{31}u_{12}+l_{32}u_{22} & l_{31}u_{13}+l_{32}u_{23}+u_{33} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & -3 \\ 1 & -2 & 5 \end{bmatrix}$$

$$\Rightarrow u_{11} = 1, \quad u_{12} = 1 \quad u_{13} = 1$$

$$l_{21}u_{11} = 3, \quad l_{31} = 1$$
$$u_{21} = 3 \qquad l_{32} = 3/2$$
$$u_{23} = -6, \quad u_{33} = 3$$

$$\Rightarrow Ly = b \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 1 & 3/2 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 10 \end{bmatrix}$$

$y_1 = 1$

$3y_1 + y_2 = 5$

$y_2 = 2$

$y_3 = 6$

eq-(v) $Ux = y$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & -6 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix}$$

$\Rightarrow$ $x_1 = 6$

$x_2 = -7$

$x_3 = 2$

$[A]_{n \times n}$

$[L]_{n \times n}$ $[\ ]_{n \times n}$

** Use mathematical induction to compute the number of elements in L & U.

Find the generalization of sum of no. of elements of L & U respectively.

** compute & Analyze the T.C for L & U req.

** Finally compute the t.c of LU factorization

$\Rightarrow$ Derive & discuss the time complexity for the soln of sys of L.eqn. using LU

The iterative process for finding the unknowns for the given set of system of linear equations -

1) Gauss jacobian
2) Gauss - seidal.

Since these are iterative process so it needs test of convergence

Test of convergence is defined as -

$$|a_{ii}| > \sum_{\substack{j=1 \\ i \neq j}}^{n} |a_{jk}|$$

here a_{ii} be the co-efficient of the diagonal elem of the corresponding system. whereas a_{jk} be the non-diagonal elements.

* the diff. b/w these two methods are for Jacobian sol^n starts from initial value ~~$a_{ij} = 0$~~ $x_{ij} = 0$; i.e.

$$x_{11} = x_{12} \cdots x_{1n} = 0$$

if x_{ij} be the ~~values~~ variables

whereas $\neq x_{ij}$ will not be zero initially or may be for ~~some~~ all other case

e.g. solve the given equation.

$$26x_1 + 2x_2 + 2x_3 = 12.6$$
$$3x_1 + 27x_2 + x_3 = -14.3$$
$$2x_1 + 3x_2 + 17x_3 = 6.0$$

1st confirm Diagonal dominance property

$$26 > |2| + |2| \quad \text{True} \checkmark$$

$$x_1 = (12.6 - 2x_2 - 2x_3)/26$$
$$x_2 = (-14.3 - 3x_1 - x_3)/3$$
$$x_3 = (6 - 2x_1 - 3x_2)/17$$

n	x_1	x_2	x_3
0	0	0	0
1	0.484	-0.529	0.352
2	0.498	-0.596	0.399
3	0.500	-0.599	0.399
4	0.500	-0.599	0.399

$x_1 = (12.6 - 0 - 0)/26$

$x_1 = 0.50 \quad x_2 = 0.59 \quad x_3 = 0.39$

e.g. of gauss seidal.

$$10x_1 - x_2 + 2x_3 = 6$$
$$-x_1 + 11x_2 - x_3 + 3x_4 = 25$$
$$2x_1 - x_2 + 10x_3 - x_4 = -11$$
$$3x_2 - x_3 + 8x_4 = 15$$

$$x_1 = (6 + x_2 - 2x_3)/10$$
$$x_2 = (25 + x_1 + x_3 - 3x_4)/11$$
$$x_3 = (-11 - 2x_1 + x_2 + x_4)/10$$
$$x_4 = (15 - 3x_2 + x_3)/8$$

n	x_1	x_2	x_3	x_4
0	0	0	0	0
1	0.6	2.32	-0.9	0.89
2	1.03	2.03	~~-1.01~~	0.98
3	1.00	2.01	1.02	0.99

x_1 → put the most updated value for x_1, x_2, x_3, x_4.
x_2 →
x_3 →
x_4 →

(HW) solve using gauss jacobian & gauss seidal. & compare the results.

$$2x_1 + 3x_2 + 6x_3 = 5$$
$$x_1 - x_2 + 3x_3 = 2$$
$$3x_1 - x_2 - x_3 = 6$$

(find x_1, x_2, x_3) & compare the results.

2 decimal

Numerical Methods - 12 (28/11/24)

* Numerical differentiation :-

Any DE wrt time t. is defined as

$$\frac{dy}{dt} = f(t, y(t)), \quad y(0) = y_0 \text{ ---(1)}$$

e.g.

$$\Rightarrow \frac{dy}{dx} = 2x + 3$$

$$\Rightarrow y = F(x)$$

According to tailor series expansion

$$y(t + \Delta t) = y(t) + \Delta t\, y'(t) + \frac{\Delta t^2}{\lfloor 2} y''(t) + \frac{\Delta^3 t}{\lfloor 3} y'''(t) + \text{---} + \infty$$

since with the higher order of Δt

$$O(\Delta^r t) \simeq 0.$$

order of $\Delta^r t$.

$$y(t + \Delta t) = y(t) + \Delta t\, y'(t) \text{ ---(11)}$$

using (1) & (11)

$$y(t + \Delta t) = y(t) + \Delta t\, f(t, y(t)) \text{ ---(111)}$$

At $x=t$, $\Delta t = h$

$$y(x+h) = y(x) + h f(x,y) \quad \text{---(iv)}$$

so,

$$y_1 = y_0 + h f(x_0, y_0)$$, at initial value of $x = x_0$, $y = y_0$

$\because \quad x_1 = x_0 + h$

$\Rightarrow x_i = x_{i-1} + h$

1st derivative

for the nth order

$$y_n = y_{n-1} + h f(x_{n-1}, y_{n-1}) \quad \text{--(v)}$$

the eqn (v) is the Euler's method (EULAR'S).

1st method: Euler's method

Method-2 (Runge-Kutta Method).

Method 1:

$$K_1 = h f(x_0, y_0)$$

$$K_2 = h f(x_0 + h, y_0 + K_1)$$

$$y_1 = y_0 + \frac{K_1 + K_2}{2}$$

Method-2

$$k_1 = hf(x_0, y_0)$$

$$k_2 = hf(x_0 + h/2, y_0 + k_1/2).$$

$$y_1 = y_0 + k_2$$

the equation (i) to (v) is the derivation of for finding the euler's method from tailor series expansion.

↳ exam :- tailor series expansion using Euler's method.

Q) find $y(0.2)$ for $y' = \frac{x-y}{2}$, $y(0) = 1$, step length $= 0.1$

$\Rightarrow$ $x_0 = 0, y_0 = 1$

$f(x,y) = \frac{x-y}{2}$

using euler's method - $h = 0.1$

$\Rightarrow$ $y_1 = y_0 + h f(x_0, y_0)$

$y(0.1) = 1 + 0.1 \cdot f(0,1)$.

$= 1 + (0.1) \times (0.5) = 0.95$

$y(0.2) = y_1 + h f(x_1, y_1)$

$= 0.95 + 0.1 f(0.1, 0.95)$

$= 0.9575$

$x_1 = x_0 + h$

$= 0 + 0.1$

$= 0.1$

$x_2 = x_1 + h$

$= 0.1 + 0.1$

$= 0.2$

Q2) $y(0.5)$, $y' = \frac{dy}{dx} = -2x - y$

$y(0) = -1$, $h = 0.1$

$\Rightarrow$ $x_0 = 0$, $y_0 = -1$, $h = 0.1$, $f(x,y) = -2x - y$

$y_1 = y_0 + h f(x_0, y_0) = -1 + 0.1 f(0,-1) = -0.9$

$y_2 = y_1 + h f(x_1, y_1) = -0.83$

$y_3 = y_2 + h f(x_2, y_2) = -0.787$

$y_4 = y_3 + h f(x_3, y_3) = -0.7683$

$y_5 = y_4 + h f(x_4, y_4) = -0.7714$.

Q3) $y(0.2)$ for $y' = \frac{x-y}{2}$, $y(0) = 1$

step length $= 0.1$

$x_0 = 0$, $y_0 = 1$

$f(x,y) = \frac{x-y}{2}$, $x_0 = 0$, $y_0 = 1$, $h = 0.1$

$\Rightarrow$ R-K Method:-① (order-1)

$k_1 = h f(x_0, y_0) = -0.05$

$k_2 = h f(x_0 + h, y_0 + h) = -0.0425$

$y_1 = y_0 + \frac{k_1 + k_2}{2}$

$= 0.953$

RK Method-② (order-2)

$k_1 = h f(x_0, y_0) = 0.05$

$k_2 = h f(x_0 + h/2, y_0 + k_1/2) = -0.045$

$y_1 = y_0 + k_2 = 0.9536$.

Numerical Methods - 13 (5/12/2024).

3rd order R-K Method -

$$K_1 = hf(x_0, y_0)$$
$$K_2 = hf(x_0 + h/2, y_0 + K/2)$$
$$K_3 = hf(x_0 + h, y_0 + 2K_2 - K_1)$$
$$y_1 = y_0 + 1/6 (K_1 + 4K_2 + K_3)$$

4th order R-K Method -

$$K_1 = hf(x_0, y_0)$$
$$K_2 = hf(x_0 + h/2, y_0 + K_1/2)$$
$$K_3 = hf(x_0 + h/2, y_0 + K_2/2)$$
$$K_4 = hf(x_0 + h, y_0 + K_3)$$
$$y_1 = y_0 + 1/6 (K_1 + 2K_2 + 2K_3 + K_4)$$

e.g: 3rd order - find $y(0.2)$, $y' = \frac{x-y}{2}$,
$x_0 = 0$, $y_0 = 1$, $h = 0.1$

$\Rightarrow$ $f(x, y) = \frac{x-y}{2}$

$\Rightarrow$ $K_1 = hf(x_0, y_0) = 0.1 \times (0.1) = -0.05$

$K_2 = hf(x_0 + h/2, y_0 + K_1/2) = -0.046$

$K_3 = h \times (x + h, y_0 + 2K_2 - K_1) = -0.042$

$y_1 = 1/6 (K_1 + 4K_2 + K_3) + y_0 = 0.9538$